VOLUME I NUMBER 4

Optica. Obs.
535.05
062
J 8

Journal
of the
Optical Society of America

Devoted to All Branches of Optics

EDITOR: H. KELLNER

ASSOCIATE EDITORS

J. A. ANDERSON
C. E. K. MEES
F. K. RICHTMYER
W. B. LANCASTER
P. G. NUTTING
HERBERT E. IVES
F. E. WRIGHT
J. P. C. SOUTHALL

JULY 1917

PAGE

An Improved Visual Acuity Test-Object 101
By HERBERT E. IVES

The Astigmatism of Lenses 108
By CHAS. W. WOODWORTH

Compound Lens Systems 113
By T. TOWNSEND SMITH

Photographic Resolving Power 119
By KENNETH HUSE

Applied Optics Abroad 134

Abstracts 134

Published Bi-monthly by the Optical Society of America, Rochester, N. Y.

OFFICE OF PUBLICATION
JOHN P. SMITH PRINTING COMPANY
ROCHESTER, N. Y.

JOURNAL

OF THE

OPTICAL SOCIETY OF AMERICA

Devoted to Theoretical, Experimental and Applied Optics

Editor: H. KELLNER,
Scientific Bureau, Bausch and Lomb Optical Co.
Rochester, N. Y.

ASSOCIATE EDITORS:

J. A. ANDERSON,
Mount Wilson Observatory
Pasadena, Cal.

HERBERT E. IVES,
United Gas Improvement Co.
Philadelphia

W. B. LANCASTER,
Ophthalmologist
Boston

C. E. K. MEES,
Research Laboratory, Eastman Kodak Co.
Rochester, N. Y.

P. G. NUTTING,
Research Laboratory, Westinghouse Co.
Pittsburgh

F. K. RICHTMYER,
Department of Physics, Cornell University
Ithaca

J. P. C. SOUTHALL,
Department of Physics, Columbia University
New York

F. E. WRIGHT,
Geophysical Laboratory
Washington, D. C.

Advertising Editor: F. K. RICHTMYER,
Department of Physics, Cornell University,
Ithaca

This Journal is the official organ of the Optical Society of America. It was founded to redeem the constitutional pledges of the society to promote the spirit of coöperation among workers in all branches of optics and coördinate the results of their labors.

To that end it publishes:

1) Original articles;
2) Short abstracts of articles of interest which have appeared in other journals;
3) Abstracts of patents;
4) The proceedings of the Optical Society of America and notes on the activities of local sections.

The Journal is issued six times a year: at the beginning of the month of January, March, May, July, September and November until further notice, and will be sent free of charge to the members of the Optical Society of America.

The Rate of Subscription is.....$5.00 per year.
Single Copies .....$1.00.

Remittances should be made to the Optical Society of America, and addressed to Adolph Lomb, Treasurer, Rochester, N. Y.

Application made for entry at the Post Office at Rochester, N. Y., as second class matter. Copyright, 1917, by Optical Society of America.

Journal

OF THE

Optical Society of America

DEVOTED TO THEORETICAL, EXPERIMENTAL
AND APPLIED OPTICS

Volume 1 JULY, 1917 Number 4

AN IMPROVED VISUAL ACUITY TEST-OBJECT

By HERBERT E. IVES

Several years ago the present writer published an account of a new apparatus for the testing of visual acuity.[1] This consisted essentially of two superposed opaque line gratings on glass, arranged to rotate relatively to each other about an axis perpendicular to their plane. When viewed by transmitted light, at such a distance that the constituent lines of the gratings are below the limit of separability, parallel dark bands are seen, due to a species of interference to be described in detail presently. These bands alter their separation gradually and continuously as the gratings are rotated, from invisibility up to extreme visibility. The average brightness of the field presented by the crossed gratings remains constant, and the variation of brightness from the center of a bright band to the center of the adjacent dark band is the same for all sizes of the bands.

These characteristic properties of crossed gratings are well known to physicists, having been employed for instance to magnify small movements of expansion and contraction, and as a regular method of testing the screws of dividing engines. The relationship holding between the width of the interference bands and the angle of relative rotation of the component

[1] *Electrical World*, April 14, 1910, p. 939.

gratings is shown by the diagram, Figure 1, where d is the distance between the centers of the ruled grating lines, θ the angle of relative rotation, and D the distance between those regions where continuous straight lines could be drawn across the gratings without crossing a clear space. These latter constitute the bands visible on viewing the crossed gratings at a distance. Their separation is given by the relation

$$D = \frac{d}{2 \sin \frac{\theta}{2}}$$

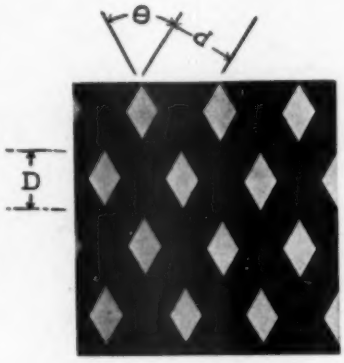


Fig. 1

In Figure 2 is shown a photomicrograph of a pair of superposed gratings, turned through a small angle, illustrating the principle. If this figure is viewed from a distance of several meters, the constituent lines disappear, and the interference bands alone show. At the distance of just visibility all indication of the graduation of intensity from center to center of light and dark bands is lost, and the bands appear merely as sharp black lines on a light field.

For use in the testing of vision the grating characteristics are correlated with visual acuity through the common measure of the latter, namely the angular separation of details just visible as separate. An observer's visual acuity will thus be proportional to $\frac{1}{D}$, or to $2 \sin \frac{\theta}{2}$. For small angles, which need alone be used by proper choice of d, this becomes $\sin \theta$, which under these conditions is simply θ. Hence the very simple relation that visual acuity can be measured with this device by the angle through which the gratings must be turned to make the interference bands just disappear.

Advantages of this form of test object over other forms, such as the test types, are the absolutely gradual transition in the size of the detail, the possibility of substituting a decimal fraction of unity for the reading in place of the present cumbersome fractions, such as $\frac{6}{12}$, etc., the fact that children and illiterates can be tested as accurately as other persons. The chief disadvantages of the test types, that different letters on the same line are unequally difficult of recognition (for instance R, B, and I), and

that their recognition depends to a considerable extent on the patient's familiarity or experience with printed letters, are eliminated by the grating test object.

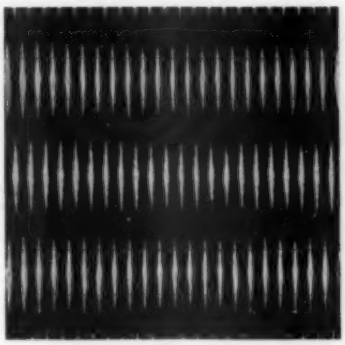


Fig. 2

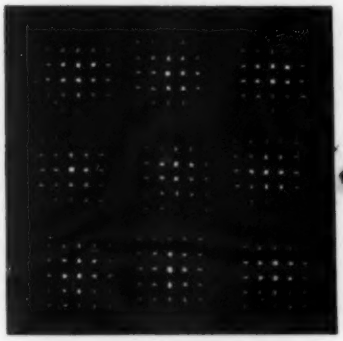


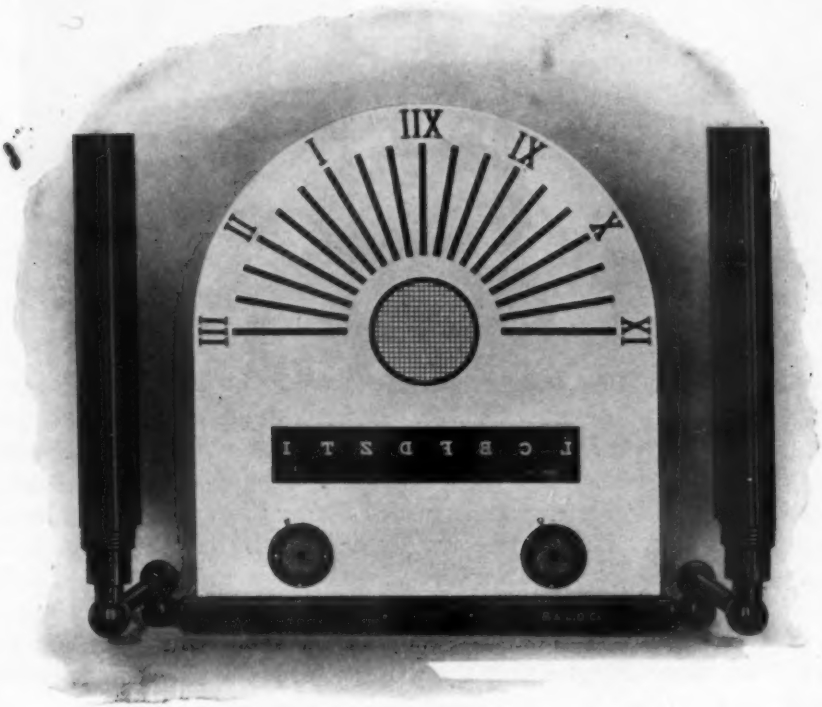
Fig. 3

Some time ago it was suggested to the writer by Mr. Max Poser, of the Bausch and Lomb Optical Co., that it would be desirable for the testing of visual acuity to have the detail of this test object extend in two directions instead of one, that is, if possible, to have squares instead of parallel bands. This improvement, which forms the principal subject of the present note, was found to be readily affected by substituting for the single line gratings, ones ruled with two sets of lines at right angles. The interference pattern due to superposing two such cross-line gratings, is shown in Figure 3, which if viewed from several meters distance, exhibits squares in the same manner that the pattern from the single line gratings in Figure 2 exhibit bands.

By the use of these cross-line gratings a measure of what may be termed the average visual acuity may be obtained. With the single line gratings, acuity may be measured at any angle, from which a measure of astigmatism is obtained. Both patterns are therefore of use, and so in the instrument as now manufactured arrangement is made for removing one of the cross-line gratings and putting in its place a single line one. This acts in conjunction with one set of lines of the remaining cross-line grating to give the bands peculiar to two single line gratings.

The apparatus as constructed and shown in Figs. 4 and 5 (front and back views), consists of a grating holder, a housing for the electric lamp which provides the illumination for the gratings, and a front plate which

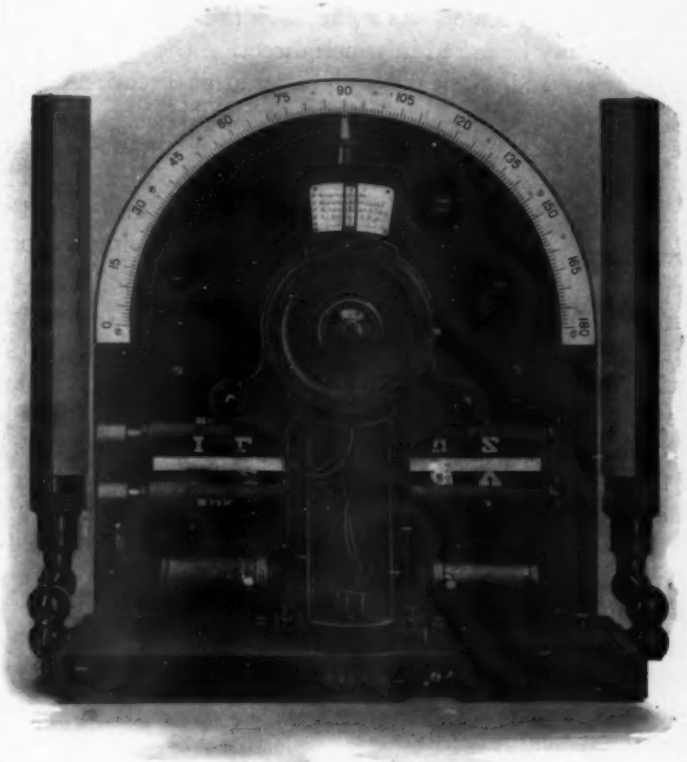
provides the surrounding field (illuminated to be of approximately the brightness of the gratings as viewed through an aperture in it), and also serves to carry various other objects for visual testing.



Visual Acuity Test Object
Front View

The grating holder, mounted directly behind the front plate, carries the two gratings so arranged that they may be rotated in opposite directions to each other by means of a rack and pinion movement, manipulated by turning a milled head on the pinion. The acuity scale is mounted just below the milled head of the rack and pinion movement and reads directly in Snellen acuity units, that is, unit acuity corresponds to the detection of the separation of lines or bands whose centers are two minutes of arc apart. This is the 5/5 or 6/6 of ordinary terminology. The ordinary 6/12 or 5/10 read as 0.5 etc. The scale is marked at the side in meters of reading

distance, indicating which line of the scale is to be used for any chosen distance between observer and test object.



Visual Acuity Test Object
Rear View, Showing Operating Mechanism

The grating holder, together with the rack and pinion arrangement, may be rotated about its axis so that the lines or squares appear at various angles to the horizontal, as indicated by a pointer on a scale of degrees on on the back of the front plate. All working parts, scales, etc., are entirely concealed behind the front plate, so that the observer need have no knowledge of what is being done by the operator.

The operation of measuring visual acuity is very simple. First the gratings are turned so that the bands or squares are too small to be visible to the observer. They are then rotated slowly by means of the rack and pinion movement until the observer just sees the pattern, whereupon the acuity is read from the scale. As a check on the correctness of his observation, the patient may be asked in which direction the lines lie with reference to the clock dial marked on the front plate, and this direction may be changed between observations so that he is in ignorance which way to expect them. Either the lines or the squares may be used for acuity testing as preferred. For finding the axis of astigmatism the lines are preferable. The gratings are to be set so that the bands are just visible and then rotated together in their holder until the bands appear clearest, when the corresponding angle is read from the scale marked on the back of the front plate.

In ascertaining the position of the axis of the cylinder with an ametoropic eye, the operator may set the test object so that the black bands coincide with the approximate position of the axis of the cylinder of the patient's eye, found by the astigmatic chart, after which the black bands of the test object are to be reduced in width by means of the rack and pinion motion of the test object and after the minimum width of the black bands just visible to the patient has been reached, the entire screen phenomenon is slightly turned around its perpendicular axis in the one and the other direction, when the position in which the black bands of the test object appear most distinct indicates the precise location of the axis of the cylinder. This process may be repeated several times and will prove that the results obtained check up very precisely. It is essential to keep the brightness of the field constant with either the phenomenon of the lines or squares and for which purpose the lamp is fitted into an adjustable sliding sleeve so that same can be moved to and from the gratings, thus either increasing or decreasing the illumination as the case may be. Interesting data may be found when the test for visual acuity is made with the gratings illuminated by more or less monochromatic light of the various colors of the spectrum. The apparatus is also useful for testing malingerers with the aid of various color screens in conjunction with the muscle test and railway signal test appliance.

Besides the special test object formed by the gratings, the apparatus as manufactured incorporates several standard devices which make the whole a very complete device for visual examinations. These are carried by

the front plate and comprise a lamp with iris diaphragm for muscle testing, special colored screens for chromatic test, lamp fitted with iris diaphragm and color screen for railway signal tests, and smoked glass wedges by means of which the effect of the colors may be dimmed or brightened as the case may be. Below the grating aperture the front plate is provided with an oblong aperture behind which two rollers are mounted carrying a set of Snellen test types, so arranged that only one row of letters is visible at one time. The test letters are white on a black ground, and three rows of each size are provided, the letters being different in each row to prevent memorizing. The Snellen test types are provided partly for the convenience of those ophthalmologists who are accustomed to their use and wish to correlate the indications of the new test object with the types. The printed letters also form excellent tests of definition, by which to judge the success of correcting glasses, although other figures such as squares, circles, or triangles would serve this purpose. After becoming accustomed to the new test object the ophthalmologist may be expected to depend entirely upon its indications, dispensing with the types.

Illumination of the front plate is by means of two long linear filament electric lamps. Both these and the grating lamp are provided with blue tinted glasses, so that the working illumination is midway between daylight and artificial light. If it is desired to operate the apparatus from the position of the patient, a mirror is used. The figures on the front dial are then reversed, and the angular scale is graduated in the direction to be correct after the reversal introduced by the mirror.

It is expected that this test object will find its first use in university laboratories, where in physical, physiological and psychological studies a satisfactory test for acuity has long been needed. Later, as students familiar with its use go into private practice it is hoped the apparatus will come into more or less general use for the everyday testing of vision.

Physical Laboratory
THE UNITED GAS IMPROVEMENT CO.
Philadelphia
July, 1917

THE ASTIGMATISM OF LENSES

By CHARLES W. WOODWORTH

A defect of the eye has probably the prior claim to the term astigmatism, but a defect in the image produced by spherical lenses has long been designated by the same word though having little in common with the former significance.

As applied to spherical lenses more than one conception of the nature of this defect prevails, making it evident that if a rigorous definition of this aberration were formulated it would be necessary to exclude some of the things which are now assembled under this term.

The best definition would seem to be the asymmetry of the wave front due to an eccentric stop, the eccentricity being in most cases relative to the object and dependent upon its distance from the optical axis.

This definition would exclude axial spherical aberration because this occurs when there is perfect radial symmetry about the optical axis. Astigmatism, therefore, is not simply the condition opposed to homocentricity. A homocentric bundle cannot show astigmatism, and a multicentric bundle if symmetric is also not astigmatic.

There is nothing in the behavior of light in spherical lenses that in any way conforms to the idea of dicentric bundles, notwithstanding that this idea is made the basis for the actual calculation of the astigmatism of lenses. The fundamental defect of Sturm's theory is the fact that in the case of oblique pencils away from the meridianal plane no refracted ray lies in the plane of the chief ray, and any equation based on the intersection of rays on such planes starts with an impossibility.

Perhaps the best way of clarifying the situation is by studying the shapes assumed by the wave front while passing through the focal region, after refraction at a single spherical surface, and taking first the case of a symmetrical bundle about the optical axis.

Figure 1 represents a succession of wave fronts at the focus formed within a dense flint glass sphere by a bundle of parallel rays. The successive outlines are sections of such wave fronts with elliptical lines added to give an approximate perspective to these figures. Each section is thickened where the light is most intense, and dotted where it is less dense than the original light.

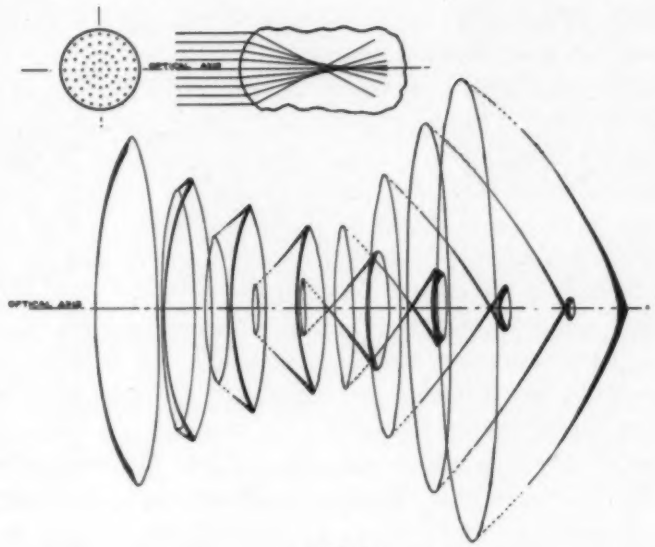


Fig. 1

Wavefronts of a pencil afflicted with spherical aberration. The section of beam before entering the refracting surface is circular and concentric with the optical axis. The insert gives a geometrical picture of the conditions.

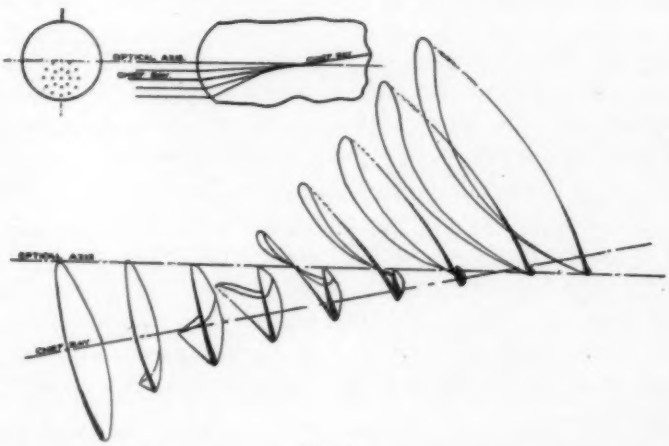


Fig. 2

Wavefronts of a pencil afflicted with spherical aberration. The section of beam before entering the refracting surface is circular and eccentric with respect to the optical axis, so that the axis of the beam strikes the refracting surface midway between center and margin of the beam as illustrated in Fig. 1. The insert shows the conditions geometrically.

A very definite series of transformations of the wave front occurs, and we may distinguish the following phases:

1. *The disk phase* with a convexity in the same direction as the lens but much less convex. The light intensity is at first very much diminished at the edges of the disk, but as the wave front approaches the focus a zone of greatest concentration is formed near the edge.

2. *The pan phase* is introduced by the inversion of the edge of the disk making a pan shaped figure with a very convex bottom. In this phase the light is most intense at the extreme edge of the bottom.

3. *The flask phase*, in which the wave front takes somewhat the shape of an Erlenmeyer flask, is brought about by the increase of the edge of the pan and the consequent narrowing of the opening. The ring of most intense light remains in the bottom of the flask.

4. *The goblet phase* is where the cup portion of goblet grows at the expense of the base till that finally disappears and a simple bowl is produced.

In case the aperture of the lens is reduced by a diaphragm the outer ends of all these curves will be eliminated, but it will be evident that while all the figures are reduced in size, still precisely the same series of transformations will occur, and the same series of symmetrical figures will result.

If, however, the diaphragm is eccentric, one side will become smaller than the other, and will go through its transformation after the other side, or if very eccentric the transformations of one side may be completely suppressed making a series of very unsymmetrical figures.

Figure 2 is so drawn that the transformation of only one side remains, that is, the diaphragm has one-half the diameter of the lens and is so placed as to reach from the center to the extreme edge. Since the behavior of the light going through the lens is not affected by the elimination of that which is prevented from entering it, the same phases may be distinguished, indeed the lines indicating the sections in this figure were traced directly from figure 1, and curves were added to give approximately perspective views of these wave fronts.

These outlines thus represent with equal accuracy the actual wave fronts of an oblique bundle of rays and exhibits the aberration we call astigmatism. The lines representing the sections are on the meridianal plane, and one will have no particular difficulty in constructing a mental

picture of a section in any other plane after a careful study of the drawing.

The straight line drawn through these figures gives approximately the position of the chief ray of this bundle, and it will take only a cursory inspection of the figure to show that the ray bears no fundamental relation to the shapes of the wave fronts in any way comparable to the relation the optical axis bears to the symmetrical figures in figure 1.

The rays which are the normals to these surfaces can be followed mentally from one figure to the next, and the following facts can be seen.

On the meridianal plane all rays on one side of the chief ray intersect this ray before any of the rays on the other side intersect it, and the extreme rays of this pencil are the farthest apart of any in their intersection on the chief ray. In other words there is no focal point on the chief ray common to any two other rays. Therefore any equation based on a common focus of the rays on the two sides is founded on an error.

On the sagittal plane lines may be drawn on the boundary of the pencil which will converge to the chief ray, but these lines are not normal to the wave fronts, and are not therefore the paths of rays of light nor are they in fact straight lines, but approach more nearly to an hyperbola intersecting obliquely an infinite number of rays. If straight lines are drawn to the chief ray forming chords of this curve, they will also intersect obliquely an infinite number of rays to which they bear no significant relation. The use of the sagittal plane of the chief ray therefore introduces into the calculation of astigmatism a false conception of the behavior of rays of light.

The only plane of the chief ray on which the asymmetry of the wave fronts can be investigated is the meridianal plane because this plane alone is normal to the wave front. There is an axis of rotation passing through the center of curvature of the lens upon which the meridianal plane may be rotated, and in every position within the limits of the lens every ray on the meridianal plane will coincide with a corresponding ray, thus making possible an exhaustive study of the details of astigmatism.

If, in the place of the sagittal plane, the conical surface made by the rotation of the chief ray were substituted, and this is the nearest approximation to the sagittal plane which is normal to the wave front, then all the rays on this surface have precisely the same focus which is at such a point that all the rays on the meridianal plane more eccentric than the chief ray

focus on it at a shorter distance, and all those less eccentric at a longer distance.

An axial bundle of light is distinguished by the fact that the chief ray coincides with an axis of curvature of the lens. The axis of rotation referred to above is the axis of curvature of the lens corresponding to the bundle, and the intersection of the oblique axis of the lens and the oblique chief ray of the bundle of rays of light is the point approximately determined by the calculation of the rays on the sagittal plane in the usual method of determining astigmatism. The axis is the line passing through this focus and the center of curvature of the lens. It also passes through the conjugate focus.

In a lens system each surface will have a separate axis, but all these axes will lie on the meridianal plane if the system is centered.

Every item of astigmatism can therefore be worked out on the meridianal plane, and for the purposes of lens calculation it is sufficient to compare the extreme ray on this plane with the chief ray to secure a true measure of the nature and the extent of astigmatism of a lens or optical system.

The astigmatism of a lens system is only slightly more complicated than the simple case here presented, and all details of it may be studied as completely as those of a single refraction on the meridianal plane in the case of a centered system.

The astigmatic surfaces are not in fact physically demonstrable phenomena, but simply surfaces mathematically locatable by connecting the intersecting points of a corresponding pair of selected rays in all possible bundles contributing to an image.

In the system of graphic lens calculation adopted by the writer, the two surfaces whose comparison is used to measure astigmatism are those produced by the intersection of the chief ray with that of the most oblique ray of the bundle, and one-half way between this and the chief ray. For coma, on the other hand, the comparision is between the surfaces of intersection of the chief ray with the most oblique and least oblique rays of all bundles. These are therefore the limiting surfaces of the focus.

AGRICULTURAL EXPERIMENT STATION
Berkeley, Cal.
July, 1917

COMPOUND LENS SYSTEMS

By T. Townsend Smith

The relation between the positions of object and image in the case of refraction at a single spherical surface is easily derived in the form

$$\frac{n_2}{u_2} - \frac{n_1}{u_1} = \frac{n_2 - n_1}{r_1} \tag{1}$$

but the problem of the formation of an image by two or more refractions is a more difficult one. The problem is one that has been treated in several ways. The solution given below seems to me to be more direct than any other one that I am familiar with.

In the case of several refractions the relation between the object and image distances from the refracting surface has the form given above for the refraction at any one of the surfaces. If, therefore, one knows the distance of an object from the first surface, one can calculate the position of the image after refraction at this surface, and using this image as the object for the second refraction and taking account of the distance between the surfaces, one can calculate the position of the image after refraction at the second surface, and so on indefinitely.

For a second spherical surface the relation between object and image distances would be

$$\frac{n_3}{u_3} - \frac{n_2}{u_2 - d_1} = \frac{n_3 - n_2}{r_2} \tag{2}$$

where, as above, distances are measured from the refracting surface, positive to the right. t_2 represents the distance between the first and the second refracting surfaces. If there are more than two refractions, for each refraction there will be a relation between object and image of the form (2). The general term may be written

$$\frac{n_{k+1}}{u_{k+1}} - \frac{n_k}{u_k - d_{k-1}} = \frac{n_{k+1} - n_k}{r_k} \tag{3}$$

and, if this be solved for u_{k+1}, the resulting expression may be written in the form

$$u_{k+1} = \frac{a_k u_k - b_k}{c_k u_k - e_k} \tag{4}$$

For the first two refractions the constants involved have the values

$$a_1 = n_2 \qquad b_1 = 0 \qquad c_1 = \frac{n_2 - n_1}{r_1} = p_1 e_1 = -n_1$$

$$a_2 = n_3 \qquad b_2 = n_3 d_2 \qquad c_2 = \frac{n_3 - n_2}{r_2} = p_2 e_2 = -(n_2 - p_2 d_1)$$

and the terms following in the case of more surfaces may be similarly expressed.

Using the form (4) and starting with the first refraction one may write

$$u_2 = \frac{a_1 u_1 - b_1}{c_1 u_1 - e_1} \tag{5}$$

and for the second refraction

$$u_3 = \frac{a_2 u_2 - b_2}{c_2 u_2 - e_2} \tag{6}$$

where the numerical values of the constants may be readily calculated from the data of any particular problem. If u_2 be eliminated between (5) and (6), the relation between u_3 and u_1 is of the form

$$u_3 = \frac{A_2 u_1 - B_2}{C_2 u_1 - E_2} \tag{7}$$

where the four constants A_2, B_2, C_2, E_2 have the values

$$A_2 = \begin{vmatrix} a_1 & c_1 \\ b_2 & a_2 \end{vmatrix} \qquad B_2 = \begin{vmatrix} b_1 & e_1 \\ b_2 & a_2 \end{vmatrix} \qquad C_2 = \begin{vmatrix} a_1 & c_1 \\ e_2 & c_2 \end{vmatrix} \qquad E_2 = \begin{vmatrix} b_1 & e_1 \\ e_2 & c_2 \end{vmatrix}$$

It is obvious that, with a third refraction, u_3 may be eliminated and an expression obtained similar to (7), but involving now u_4 and u_1 alone. If, then, the constants are evaluated, there will be a simple relation of the form (7) between the object and image distances from the first and last refracting surfaces respectively. This process may be repeated as often as is necessary, giving an expression of the form (7) connecting u_n and u_1 and involving constants the values of which are perfectly definite and may be readily calculated. This relation by a suitable change of variables may be reduced to the form of (1) above.

Equation (7) represents a rectangular hyperbola. Writing u for u_1 and v for u_n and leaving off the subscripts in the constants, one has after n refractions.

$$(8) \qquad \left(u - \frac{E}{C}\right)\left(v - \frac{A}{C}\right) = u'\,v' = \frac{AE - BC}{C^2}$$

Here E/C and A/C are obviously the values that are assumed by u and v when the other is infinitely great, and hence give the points to which parallel light will be converged by the optical system. The hyperbola in question lies in the second and fourth quadrants, inasmuch as in the case of refraction the quantity on the right in (8) is negative. [1]

The constant term (B) in the form (7) may be eliminated and the expression reduced to the form for a single refraction (1) by any transformation of coördinates that will make a point on either branch of the hyperbola the origin of coördinates. In other words, if any position that may be assumed by an object and the corresponding position for the image of that object be used as the zero points for measuring the distances u'' and v'' to any other object and image, the relation between these distances takes the form of (1). It is possible, therefore, to reduce the case of many refractions to the case of a single refraction in an indefinite number of ways. The choice of the particular pair of conjugate points from which our distances shall be measured is to be governed by convenience.

In the case of a single refraction a refracted ray intersects itself at the refracting surface. If, therefore, an object is immediately in front of the refracting surface, its image is immediately back of the refracting surface and of the same size as the object, or, in other words, the linear magnification is one. The case of many refractions may be made almost exactly like the case of a single refraction if the respective object and image distances are measured from the pair of conjugate points for which the magnification is one. These points are called the "unit" or "principal" points, and the planes perpendicular to the axis of the system at these points the "unit planes" or the "principal planes."

It is easy to show that in the case of refraction at a spherical surface, the magnification is[2]

$$(9) \qquad \frac{y_2}{y_1} = \frac{n_1 \tan \omega_1}{n_2 \tan \omega_2}$$

where y_2 and y_1 represent the height of image and object respectively, and ω_1 represents the angle (assumed small) made with the axis by any ray leaving the axial point of the object, and ω_2 represents the angle made by

[1] Houstoun—Treatise on Light, p. 36.

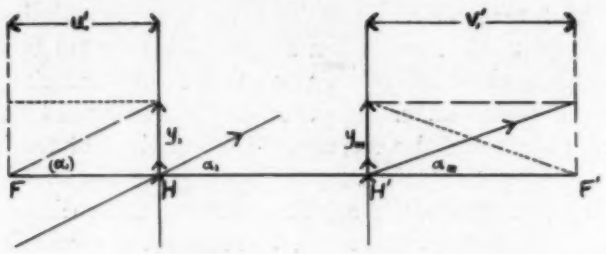
[2] Cf. Heath, Houstoun, Southall, or any treatment of geometrical optics.

this ray with the axis after refraction. This relation would hold after any number of refractions (m), for a similar relation holds between y_3 and y_2, so that both y_2 and ∞_2 may be eliminated by multiplying the two equations together.

Applying now the condition that y_1 is to be equal to y_m, the result is

(10) $$n_1 \tan \infty_1 = n_m \tan \infty_m$$

or, as the accompanying figure shows,



(11) $$n_1 \frac{y_1}{u_1'} = - n_m \frac{y_m}{v_1'}$$

which gives

(12) $$n_1 v_1' = - n_m u_1'$$

as the relation between u_1 and v_1 in the case of unit magnification. This condition together with (8) gives the distances from the principal focal planes to the unit planes, and the negative of these distances will be the principal focal lengths of the system referred to the unit points as zero points.

The solution for the principal focal lengths gives

(13) $$- u_1' = f = \mp \sqrt{\frac{n_1}{n_m}\left(\frac{BC - AE}{C^2}\right)} - v_1' = f' = \pm \sqrt{\frac{n_m}{n_1}\left(\frac{BC - AE}{C^2}\right)}$$

The distance from the first refracting surface to the first principal plane is

(14) $$L = \frac{E}{C} - f$$

and from the last refracting surface to the second principal plane

(15) $$L' = \frac{A}{C} - f'$$

The distances from the principal points to the nodal points may be worked out in ways that suggest themselves.[3]

[3] Consult, e. g., Drude—Theory of Optics, p. 23 (Art. 23).

The Calculation of the Constants

The constants in the equation

$$u_{m+1} = \frac{A_m u - B_m}{C_m u_m - E_m} \tag{16}$$

representing the case of m refractions may be written in determinant form.

For the case of two refractions, the thick lens, the expressions are given following equation (7) above, and for the case of a thick lens the constants of the system are given following (4). The general expression for the n-row determinant may be built up from these values. These give

$$A_m = \begin{vmatrix} a_m & b_m \\ C_{m-1} & A_{m-1} \end{vmatrix} \quad B_m = \begin{vmatrix} a_m & b_m \\ E_{m-1} & B_{m-1} \end{vmatrix} \quad C_m = \begin{vmatrix} c_m & e_m \\ C_{m-1} & A_{m-1} \end{vmatrix} \quad E_m = \begin{vmatrix} c_m & e_m \\ E_{m-1} & B_{m-1} \end{vmatrix}$$

Building up in this way the values after three refractions are:

$$A_3 = \begin{vmatrix} a_1 & c_1 & 0 \\ b_2 & a_2 & b_3 \\ e_2 & c_2 & a_3 \end{vmatrix} \quad B_3 = \begin{vmatrix} b_1 & e_1 & 0 \\ b_2 & a_2 & b_3 \\ e_2 & c_2 & a_3 \end{vmatrix} \quad C_3 = \begin{vmatrix} a_1 & c_1 & 0 \\ b_2 & a_2 & e_3 \\ e_2 & c_2 & c_3 \end{vmatrix} \quad E_3 = \begin{vmatrix} b_1 & e_1 & 0 \\ b_2 & a_2 & e_3 \\ e_2 & c_2 & c_3 \end{vmatrix}$$

After six refractions one has:

$$A_6 = \begin{vmatrix} a_1 & c_1 & 0 & 0 & 0 & 0 \\ b_2 & a_2 & b_3 & e_3 & 0 & 0 \\ e_2 & c_2 & a_3 & c_3 & 0 & 0 \\ 0 & 0 & b_4 & a_4 & b_5 & e_5 \\ 0 & 0 & e_4 & c_4 & a_5 & c_5 \\ 0 & 0 & 0 & 0 & b_6 & a_6 \end{vmatrix} \qquad B_6 = \begin{vmatrix} b_1 & e_1 & 0 & 0 & 0 & 0 \\ b_2 & a_2 & b_3 & e_3 & 0 & 0 \\ e_2 & c_2 & a_3 & c_3 & 0 & 0 \\ 0 & 0 & b_4 & a_4 & b_5 & e_5 \\ 0 & 0 & e_4 & c_4 & a_5 & c_5 \\ 0 & 0 & 0 & 0 & b_6 & a_6 \end{vmatrix}$$

$$C_6 = \begin{vmatrix} a_1 & c_1 & 0 & 0 & 0 & 0 \\ b_2 & a_2 & b_3 & e_3 & 0 & 0 \\ e_2 & c_2 & a_3 & c_3 & 0 & 0 \\ 0 & 0 & b_4 & a_4 & b_5 & e_5 \\ 0 & 0 & e_4 & c_4 & a_5 & c_5 \\ 0 & 0 & 0 & 0 & e_6 & c_6 \end{vmatrix} \qquad E_6 = \begin{vmatrix} b_1 & e_1 & 0 & 0 & 0 & 0 \\ b_2 & a_2 & b_3 & e_3 & 0 & 0 \\ e_2 & c_2 & a_3 & c_3 & 0 & 0 \\ 0 & 0 & b_4 & a_4 & b_5 & e_5 \\ 0 & 0 & e_4 & c_4 & a_5 & c_5 \\ 0 & 0 & 0 & 0 & e_6 & c_6 \end{vmatrix}$$

The law of the growth of the determinant is clear enough from the examples given to make unnecessary the writing down of the general term, which could easily be done. It will be in general observed, as the six row determinant shows, that only the outside rows and columns change as one passes from one constant to another.

Another Method for Calculating the Constants

The following more convenient method of calculating the constants for the case of several refractions was suggested to me by Professor Lefschetz of our Department of Mathematics.

The expression

$$u_2 = \frac{a_1 u_1 - b_1}{c_1 u_1 - e_1}$$

may be represented symbolically as

$$u_2 = \begin{pmatrix} a_1 & b_1 \\ c_1 & e_1 \end{pmatrix} u_1$$

which is to be considered merely as a short-hand notation for the longer expression. Using this notation one may write

$$u_3 = \begin{pmatrix} a_2 & b_2 \\ c_2 & e_2 \end{pmatrix} \qquad u_2 = \begin{pmatrix} a_2 & b_2 \\ c_2 & e_2 \end{pmatrix}\begin{pmatrix} a_1 & b_1 \\ c_1 & e_1 \end{pmatrix} u_1$$

where the double expression is to be interpreted as

$$u_3 = \begin{pmatrix} a_2 a_1 - b_2 c_1 & a_2 b_1 - b_2 e_1 \\ c_2 a_1 - e_2 c_1 & c_2 b_1 - e_2 e_1 \end{pmatrix} u_1$$

and this expression may be written in the expanded form as

$$u_3 = \frac{(a_2 a_1 - b_2 c_1) u_1 - (a_2 b_1 - b_2 e_1)}{(c_2 a_1 - e_2 c_1) u_1 - (c_2 b_1 - e_2 e_1)}$$

The rule for repeating the operation is now apparent. It is to multiply the terms in a row by the corresponding terms in a column in order to obtain the term in the position common to both row and column.

For three refractions, one writes

$$u_4 = \begin{pmatrix} a_3 & b_3 \\ c_3 & e_3 \end{pmatrix}\begin{pmatrix} a_2 & b_2 \\ c_2 & e_2 \end{pmatrix}\begin{pmatrix} a_1 & b_1 \\ c_1 & e_1 \end{pmatrix} u_1$$

$$= \begin{pmatrix} a_3 & b_3 \\ c_3 & e_3 \end{pmatrix}\begin{pmatrix} a_2 a_1 - b_2 c_1 & a_2 b_1 - b_2 e_1 \\ c_2 a_1 - e_2 c_1 & c_2 b_1 - e_2 e_1 \end{pmatrix} u_1$$

$$= \begin{pmatrix} a_3(a_2a_1 - b_2c_1) - b_3(c_2a_1 - e_2c_1) & a_3(a_2b_1 - b_2e_1) - b_3(c_2b_1 - e_2e_1) \\ c_3(a_2a_1 - b_2c_1) - e_3(c_2a_1 - e_2c_1) & c_3(a_2b_1 - b_2e_1) - e_3(c_2b_1 - e_2e_1) \end{pmatrix} u_1$$

and form refractions

$$u_{m+1} = \begin{pmatrix} a_m & b_m \\ c_m & e_m \end{pmatrix}\begin{pmatrix} a_{m-1} & b_{m-1} \\ c_{m-1} & e_{m-1} \end{pmatrix} - - - - - - \begin{pmatrix} a_1 & b_1 \\ c_1 & e_1 \end{pmatrix} u_1$$

This may be expanded, following the rule above, in any given case. This expansion will give the constants A_m, B_m, C_m and E_m. From these the cardinal points of the refracting system may be obtained as indicated above.

Thin Lenses

It is obvious that in the case of a set of thin lenses, the relation between distances to object and image is of the form of (4), and that the method of attack herein outlined will apply to such a case without change.

Blake Physical Laboratory
University of Kansas
September, 1917

PHOTOGRAPHIC RESOLVING POWER[1]

By Kenneth Huse

Photographic resolving power is generally defined in terms of the distance by which two minute images, lying adjacent to one another, are just separated in the developed plate. In the judgment of the separation it is assumed that the image is so viewed as to be well within the resolving power of the eye and also not magnified to an extent that will make the individual silver grains more apparent than their groupings. This is a logical definition, strictly in accord with that of the resolving power of optical instruments, but it should be pointed out that the measurement of photographic resolving power is not an angular measurement but a linear one. From these considerations it is seen that the photographic records of spectral lines and double stars are extreme tests of the resolution of the photographic plate, since the developed image must show separate lines in the one case and double stars in the other. The inhomogeneity of the photosensitive surface prohibits the production of perfectly sharp pictures. This is due to the manner in which the silver image is broken up by the grains composing it. That is to say, that since the photographic surface is a highly diffusing medium, light striking it is scattered in all directions. This irradiation or spreading of the image is due either to the light diffusely reflected in the film or to diffraction in extremely fine grained plates.

The first consideration of this subject was given by Wadsworth[2] who defined photographic resolution, the linear distance between the centers

[1] Communication No. 61 from the Research Laboratory of the Eastman Kodak Co.
[2] *Astrophysical Journal*, Vol. 3, 1896, pp. 188 and 321.

of two lines to be just separated, as equal to four times the diameter of the silver grain, designated as "e." E. C. C. Baly[3] stated that "e" varied for different brands of plates but that it may be taken as lying between 0.005 mm and 0.025 mm. This does not agree, however, with the values obtained by Mees and Sheppard,[4] who give them as varying from 0.001 mm to 0.0034 mm. But even assuming the latter value to be correct, the 4 "e" of Wadsworth's formulation would give values of the resolving power which are far greater than any found in practice. Mees[5] took up the subject experimentally, studying the images produced by photographing an illuminated spectroscopic slit in a reducing camera. The camera was fitted with a highly corrected telescopic objective working up to its computed optical resolving power. The developed image of this linear source, thus projected by the lens upon the photographic plate, was found to have spread beyond its true optical boundaries. Mees showed that although this irradiation did decrease with decrease of grain size, since the light scattered in a coarse grained plate is quite considerable, that after a certain state of fineness was passed, spreading again set in. This he considers as due to diffraction at the edges of the silver grains. With the substitution of a grating of known periodicity for the slit test object, Mees was able to test the ability of various plates to separate objects lying adjacent to one another, and as the grating constants and the degree of reduction were known, a value of resolving power could be computed. His results were that for a fine grained emulsion, lines will be just resolved if they are separated by 0.018 mm and for a Speed emulsion of the coarse grain type, a separation of 0.030 mm between lines is necessary for resolution. With regard to the effect of light of different colors upon photographic resolution, Mees first found that resolving power to red light was higher than that attained by exposure to the blue, but later agreed with Scheffer, that resolution was equal for both red and blue light, stating that the former discrepancy was probably due to the presence of chromatic aberration in the objective.

Scheffer[6] was the next investigator in this field of photographic research. Although employing an entirely different type of camera from that of Mees, the general scheme was also the photographing of a grating of

---

[3] *Spectroscopy*, E. C. C. Baly (Longman, Green).
[4] *Investigation of the Photographic Process* (Longman, Green).
[5] Proc. Royal Soc. (A) Vol. 83, p. 10, 1909.
[6] *British Journal of Photography, 1910.*

known periodicity and determining the extent to which the plates resolved it. He introduced a new term "turbidity" as descriptive of the scattering properties of an emulsion. The results of Scheffer's investigation show also that resolution is dependent on the amount of irradiation displayed by the emulsion and that length of exposure, time of development and the wave length of the incident light affect resolving power.

Since lens-produced images are in no case exact reproductions of the object, it is obvious that the foregoing methods could never give absolute information concerning photographic resolving power. Féry and Scheiner in some of the earliest investigations on resolution did not employ lenses in making their tests; Goldberg[7] was the first investigator of recent years to dispense with the lens method in obtaining exposures. Goldberg's scheme was to study the spreading of the image by examination of discs produced upon the photographic plate by exposure through a metal plate pierced with a series of conical holes, which plate was in intimate contact with the photographic plate during exposure. He gave exposures ranging from 10 to 100,000 times the "Schwellenwert," or threshold value, of the plates investigated. From the results of his work he stated that the study of photographic resolving power divides itself into two lines of investigation, the estimation of turbidity and the determination of the resolving limit of the plate or film.

Nutting[8] pointed out that the image distribution (in the developed film) could be best studied along a straight-edge. This he accomplished by laying an accurately sharpened straight-edge in contact with the emulsion and exposing to parallel monochromatic light. The images thus obtained show the manner in which the light has spread under the edge which was in intimate contact with the plate. This shading off of density in the developed image from the true position of the edge is termed "density gradient," and Nutting deduced mathematically that density gradient is equal to the product of the photographic gradient and the light gradient. That is to say, the density gradient, or the rate of spreading of the image, is equal to the contrast, that is, the degree of development multiplied by the turbidity.

Tugman[9] determined the influence of exposure, time of development and the wave length of the incident light on density gradient. His results, using the straight-edge, on the effect of exposure, checked Nutting's, namely,

[7] *Photographic Journal*, 1912.
[8] *Photographic Journal*, 1914.
[9] *Astrophysical Journal*, Vol. 42, p. 331, 1915.

that no appreciable spreading away from the edge of the image could be detected in cases of densities less than unity (that is, deposits transmitting 1/10 of the incident light). With further increase in exposure shading off from the edge became noticeable, but at a density of 2 (a transmission of 1/100th) the gradient reached a fixed value which was altered little by further increase in exposure. Tugman found that prolonged development decreased density gradient. This he attributed to the manner in which the developer penetrated into the emulsion. Tugman also found that resolving power for light of short wave length was greater than for light of the longer wave lengths.

The object of this investigation was to determine the influence of specific factors, exposure, development and wave length of light on photographic resolving power as found by the grating test.

The method devised by Mees was employed in this investigation and a brief description is given, including some slight modifications. The apparatus consists essentially of a reducing camera, arranged to insure an exact focal plane, in the form of a metal tube 20 feet long and of 4 inches diameter, with the interior thoroughly blackened. At one end of the tube are the light source and an arrangement for holding the test object, while at the other a camera is built with a plate holder carrier attached. The light source employed was a seasoned 25 watt concentrated tungsten filament stereopticon lamp, burning at about 1.35 watts per candle. A voltmeter, introduced lnto the circuit, controlled the light source, maintaining constant illumination. The lamp was enclosed in a white-walled parabolic reflector over the front of which were placed diffusing media to insure uniform illumination of the test object. The camera was provided with a highly corrected Fuess telescopic objective of F 5.3 aperture ratio and of 15.29 $\pm$ 0.01 cm focal iength. Focus was obtained by means of a slow motion screw.

The test object, Fig. 1, a fan shaped converging grating consisting of alternating light and dark sectors each about 1.5 degrees in angle, is photographed on the sensitive material and the minute image thus obtained, only one millimeter long, is examined under a micrometer microscope to ascertain the distance from its apex the image is resolved. Now, since the spacings of the grating and the scale of reduction are known, a numerical expression

Fig. 1

for resolving power is found. This numerical value of resolving power thus deduced is best illustrated by an example. For instance, a value equal to fifty, simply means that the plate or film under the conditions of the experiment is capable of resolving fifty lines to the millimeter, measured from center to center, similarly, a value of one hundred indicates that one hundred lines to the millimeter are resolved. While this method introduces systematic errors which prevent it from being suitable for a final investigation, data derived by it will be useful as a means of comparison with the edge gradient in an endeavor to determine the relation between sharpness of images and resolving limit. This work is now being studied further.

The errors involved in this method of study of photographic resolving power are due to the use of a lens and to the fact that the measurement of resolution introduces the personal equation to a certain extent. That these errors are really small under the conditions of these experiments is evident from the following. The residual aberrations of the highly corrected objective used, are negligible, on the axis, and the only other source of error would be that the resolving power of the lens might be comparable with that of the plate. That this is not true, is evident from the following considerations. Nutting[10] formulates optical resolving power as $\frac{\delta}{\lambda} = a\frac{f}{r}$ where λ is the wave length of the light (in this case 0.00048 mm., approximately the wave length of the maximum sensitiveness of the material) a, is a constant and equal to 0.5 for a just resolvable image, and $\frac{f}{2r}$ is the aperture ratio (*f*: 5.3 in this case). The value thus obtained is 395, and since the highest value of photographic resolving power found in this investigation was 125, it is obvious that the resolving power of the lens introduces no measurable error. As regards the personal equation, this factor was greatly minimized by the fact that all readings on the microscope and hence the computed values of photographic resolving power are the mean of several determinations. An actual case showing the deviation of the individual values from the mean is shown in the following table:

[10] *Outline of Applied Optics*, P. Blackison's Son & Co., 1912.

Values of Resolving Power.

69.0
70.0
69.6
70.5
69.3
71.0
68.0
68.3
70.4
70.4

Mean Value = 69.5

Maximum Deviation = 1.5 or approximately 2.2%.

Theory shows that there are three main elements entering into the problem of photographic resolving power, which have been called turbidity, contrast and opacity. Turbidity may be defined as the scattering power of an emulsion. Contrast is the sensitometric constant "gamma," and is therefore dependent upon the time of development, constitution of the developing solution, the type of plate and the wave length of the incident light. Opacity is a factor in the determination of the extent to which turbidity acts, since scattered light can not travel far in an opaque medium.

Resolution is dependent upon the irradiation or, that is to say, the turbidity of the emulsion, and since this factor bears a relation to grain size, resolution is therefore a function of grain. In general resolving power decreases with increasing size of grain, since the larger the grain the more light is scattered due to reflection, and the less the plate can resolve, but with plates of a grain size of the order of the half wave length of light the resolution will be affected by diffraction of light at the edges of the grain. The following table shows the general trend of this effect. Here the brand of plate and its average H. and D. speed value are given, together with the value of resolving power obtained. For reasons that will be obvious later, all plates were given a critical exposure and developed to the same degree in the same developer (pyro) so as to make the comparison a more nearly fair one. Of course, the albumen plate was developed physically, not chemically. The table is as follows:

PLATE	H. AND D. SPEED	RESOLVING POWER
Albumen	0.01	125
W. and W. Resolution	3.0	81
W. and W. Slow Process Pan	5.0	67
Seed Lantern (Yellow Label)	6.0	62
Positive Motion Picture Film	10.0	42
Seed 23	150.0	35
W. and W. Panchromatic	200.0	31
Seed 30	400.0	29
Seed Graflex	450.0	25

Seed lantern plates were selected as the material to be used in the remainder of the investigation so that any reference to sensitive materials will be understood to be this medium unless otherwise stated.

Experiments were conducted to ascertain the variation in resolution due to the length of exposure, the time of development being maintained constant. Thus exposures were given to the plate increasing logarithmically in the range from over to under-exposure. This range is best explained by reference to the characteristic curve of a photographic emulsion given in Fig. 2. This curve shows the relation existing between the density and the logarithm of the exposure. It is seen that through a considerable portion of the curve the density is proportional to the logarithm of the exposure, that is, through this portion the range of light intensities in the original is correctly rendered by the opacities of the negative. The lower part of the curve is termed the period of under-exposure, the density increasing less rapidly than the logarithm of the exposure, and the top of the curve is termed the period of over-exposure, where the density fails to rise in the correct proportion. A strip of plate having received exposures in this range was developed at a constant temperature in a slightly bromided Pyro developer for three minutes and the resultant images measured. The computed values of resolution were then plotted against the logarithm of

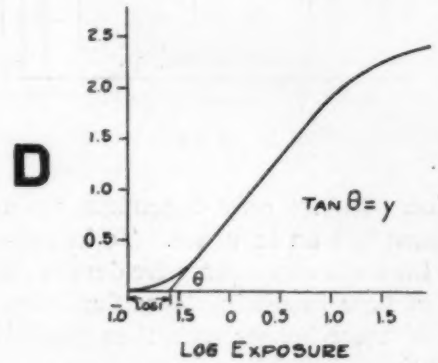


Fig. 2

the exposure given and as can be seen from Fig. 3, there is a decidedly critical value where the best resolution is manifested. It is evident from this curve that not only is over-exposure detrimental to resolution but so also is serious under-exposure.

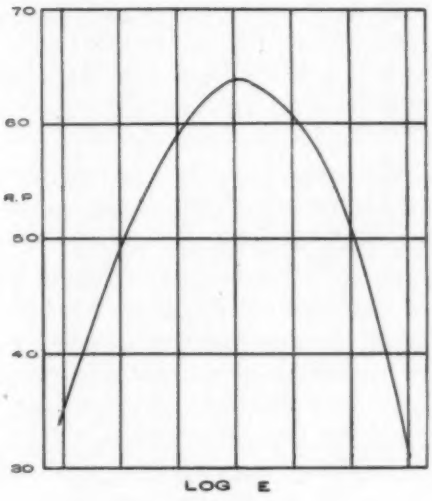


Fig. 3

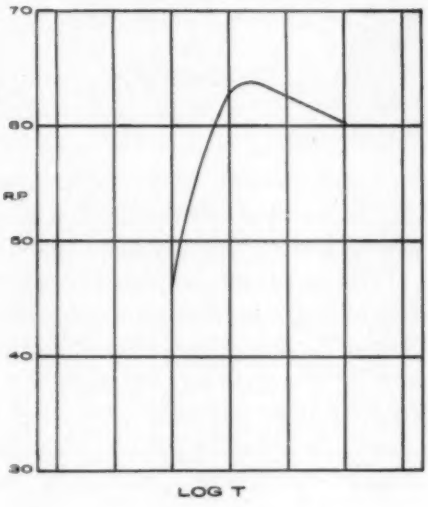


Fig. 4

Scheffer pointed out that resolution was not only dependent upon exposure, but that the time of development had an influence. To investigate this, several strips, having received the same exposures, were developed at a constant temperature for periods of time increasing logarithmically. The values derived from measurement of these images were then plotted against the logarithm of the developing times and Fig. 4 shows that the highest resolution is not attained with short development, and that this is also the case with great contrast, there being an optimum time for the best resolution. This optimum time of development is such as to produce a contrast of unity, i. e. from Fig. 2, $\tan \theta = 1.0$, so that it seems that correct exposure, that is, an exposure such that the densities lie on the straight line portion of the curve, and average contrast yield highest resolving power.

Another experiment shows the effect of exposure and development together on resolution. Fig. 5. Attention is called to the fact that the

maximum resolving power seems to be obtained with slight over-exposure and short development, the contrast, however, is such as to make it of little use. A constant decrease in maximum resolution is noted with normal or correct exposure and development, as previously stated.

Now it has been shown that photographic resolving power is extremely sensitive to length of exposure and time of development and that it also

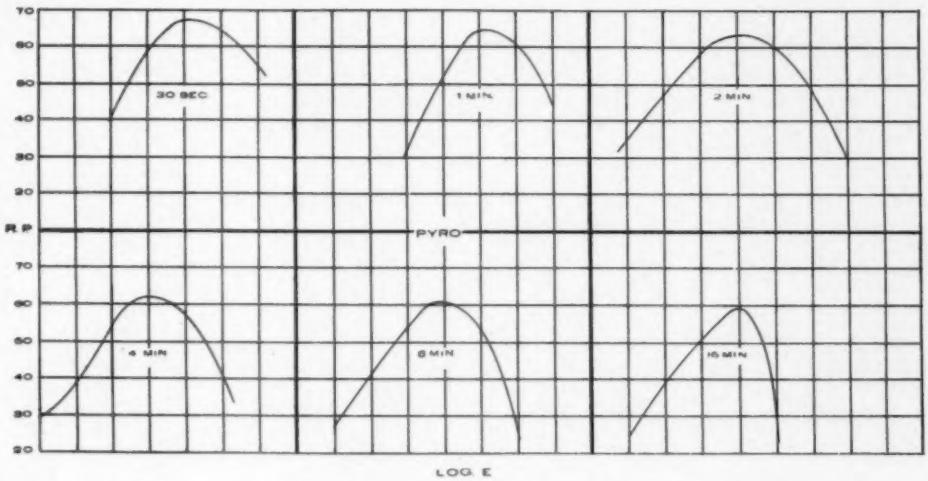


Fig. 5

depends greatly upon the type of emulsion used. There is another decidedly important factor that it seems safe to predict will have great bearing upon resolution and that is the reducer or developer employed.

These experiments were carried out only on the one material, this being the same as that previously used, so that the results apply solely to it, but a means of comparison is afforded which may prove useful. In order to determine any variation of resolution with the reducer on a given material, investigations similar to those just outlined were carried out with twenty (20) other developers. The data derived from these experiments are summarized in the following table:

TABLE I.

DEVELOPER	MAXIMUM RESOLVING POWER	EXPOSURE	DEVELOPMENT
Pyro Caustic	77.0	4 sec.	2 min.
Glycin	69.0	3 "	1 "
Hydrochinon	64.0	3 "	2 "
Pyro	64.0	3 "	2 "
MQ 25	64.0	3 "	2 "
Metol	63.0	3 "	2 "
Nepera	62.0	3 "	2 "
Pyrocatechin	62.0	8 "	2 "
Pyro Metol	62.0	8 "	2 "
Eikonogen-Hydrochinon	61.0	4 "	3 "
Ferrous Oxalate	61.0	2 "	4 "
Caustic Hydrochinon	57.0	4 "	2 "
Eikonogen	57.0	4 "	4 "
Kachin	54.0	2 "	5 "
Amidol	51.0	2 "	4 "
Process Hydrochinon	50.0	2 "	2 "
Ortol	49.0	4 "	2 "
Rodinal	49.0	8 "	2 "
X-Ray Powders	49.0	3 "	1 "
Edinol	47.0	4 "	16 "

From this table it is seen that resolving power is not an inherent property of the plate or film but is very dependent on the reducing agent employed in development. Thus, on the material used (Seed Lantern) in these experiments, resolving power values were obtained varying from 47 to 77 by simply altering the developer. That is to say, that in the one case the plate is able to resolve 77 lines to the millimeter, whereas in another only 47 lines are resolved. The values of exposure and time of development tabulated above necessarily apply only to the particular conditions of these experiments, serving merely to show the dependence of good resolution on these factors as well as their variation with the reducer or developer used. An attempt was made to evolve some relation between this ability of a developer to give good resolution and its reduction potential, but none was obtained.

The curves of Fig. 6 show the variation in resolution obtained with exposure with the various developer used. The curves are, in general, the same, with possibly one or two exceptions. It will be noted that the point

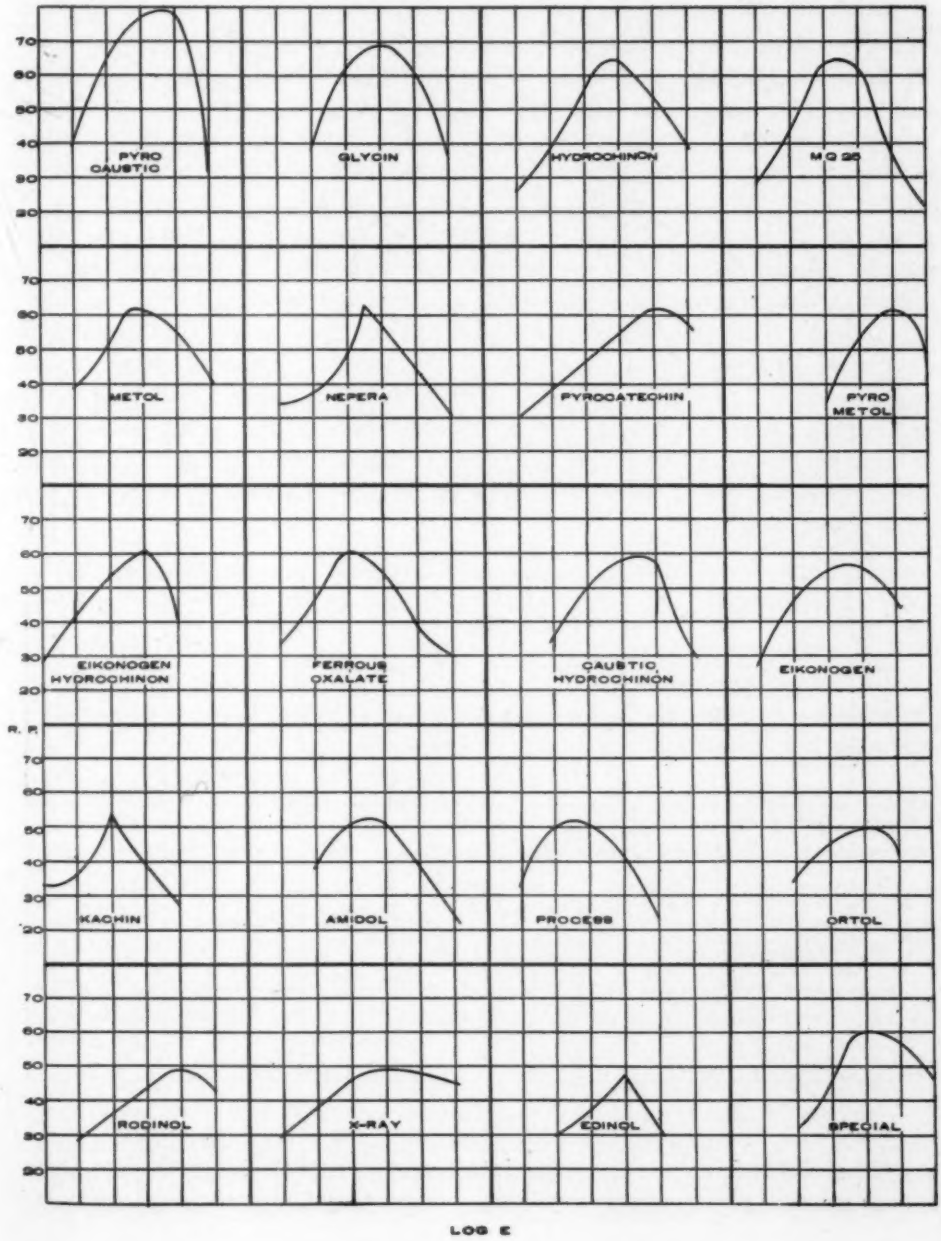

PYRO CAUSTIC
GLYCIN
HYDROCHINON
M Q 25
METOL
NEPERA
PYROCATECHIN
PYRO METOL
EIKONOGEN HYDROCHINON
FERROUS OXALATE
CAUSTIC HYDROCHINON
EIKONOGEN
R. P.
KACHIN
AMIDOL
PROCESS
ORTOL
RODINOL
X-RAY
EDINOL
SPECIAL
LOG E

Fig. 6

at which highest resolution is obtained, with the use of Nepera and Kachin, in particular, is very critical, thus showing that resolving power is subject to delicate changes in exposure under certain conditions. A striking example of an altogether opposite effect is given by the curve of the Process developer where, although maximum resolution is not high, a constant value is obtained over quite a range of exposures, thus exhibiting a considerable degree of latitude. A similar effect, but not to such a marked extent, is shown by the curves of Caustic Pyro, Hydrochinon, MQ 25, Metol and some others. Thus, not only is resolution dependent on the exposure, but the extent of this dependence is related to the developer used.

The time of development, that is, the contrast to which the plate is developed in the various solutions, is of interest because of some special cases found. In general, however, with normal exposure, resolution is, as has been stated, not highest for short development and great contrast is detrimental to good resolution, but between these limits it is fairly uniform. The curves of Fig. 7 show the changes due to contrast in some of the more interesting cases.

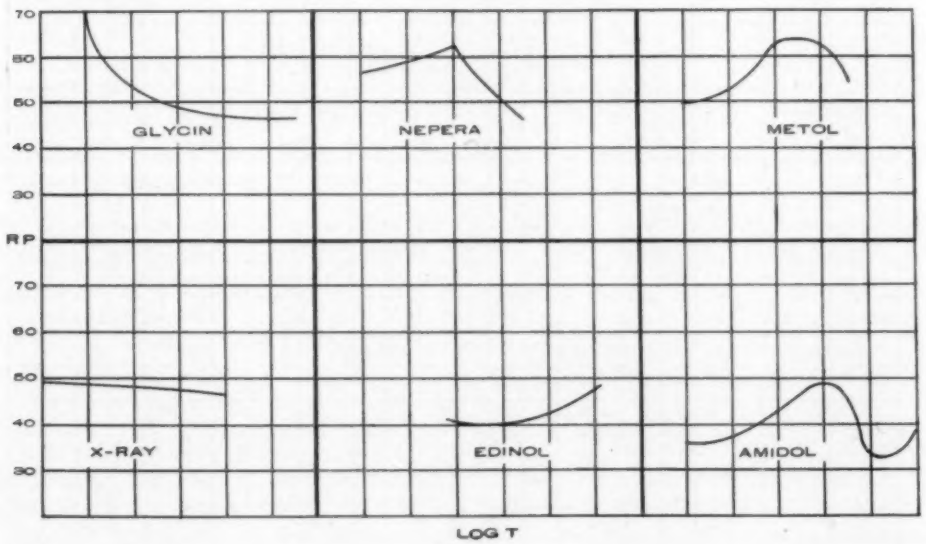


Fig. 7

The plates from which these data were derived in order to plot these curves were all given normal exposure. The normal case of the variation of

resolution with contrast is represented by the curves of Metol or Nepera. Here the curves show maxima where best resolution is obtained and it is to be noted that resolution falls off rapidly with over-development as with over-exposure. An interesting case is shown by the curve of Glycin where best resolution is evidenced on short development, there being a drop to a constant but considerably lower value with prolonged development. The case of the X-Ray developing powders, which consist of an elon-hydrochinon mixture, the formula being so arranged as to give great penetration, shows constant but rather low resolving power over quite a range of development times. A peculiar effect was noted in the experiments with the use of Amidol. From the curve it is seen that with under and correct development this developer behaves normally, but with over-development there is a decided increase in resolution following the usual decrease as in the normal case. That is to say, on the fan image the sector edge, on development to high contrast, was cleared from the usual fog at an area immediately surrounding it, thus this lightening near the edge produced better resolution. It is suggested that this effect may be due to an exhaustion of the developer. Still another type is represented by Edinol, in which case maximum resolution is obtained with high contrast, that is, with prolonged development.

From these results it is evident that the resolving power of a photographic plate is dependent on many factors: the exposure, the time of development, and the developer used, as well as the type of emulsion. It has also been shown that the effect of exposure and contrast are dependent greatly on the reducer employed.

It is of interest to note the effect of light of different colors on photographic resolving power. Mees found that an ordinary fine grained plate resolved less when exposed to blue light than when the plate was bathed in pinacyanol and given an exposure to red light. Scheffer, however, conducted experiments which led him to conclude that photographic resolution in an ordinary plate exposed to the blue end of the spectrum was equal to that of the same material bathed in pinacyanol and then subjected to the red end. Mees later agreed with Scheffer, concluding that his previous values might have been influenced by the presence of chromatic aberration in the lens employed. In the following experiment, the three types of photographic emulsions, ordinary, orthochromatic and panchromatic, whose spectral sensitivities are depicted in Fig. 8, were tested. Wratten and Wainwright monochromatic filters were used, Numbers 50, 63 and 70 were used. Filter

number 50 is a monochromatic blue having maximum transmission of thirteen per cent at wave length of 555 $\mu\mu$ and decreasing to a negligible amount at 430 $\mu\mu$ and 480 $\mu\mu$. Filter number 63 is a monochromatic green having a maximum transmission of seven per cent at 530 $\mu\mu$ and decreasing to

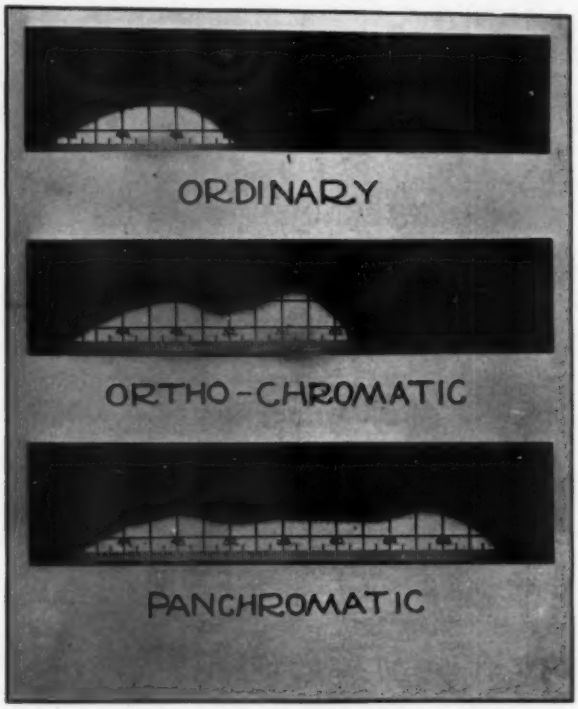


Fig. 8

practically zero at 500 and 570 $\mu\mu$. Filter number 70 is a monochromatic red having a maximum transmission of forty-five per cent at 700 $\mu\mu$ and decreasing to four-tenths of a per cent at 650 $\mu\mu$. With the ordinary slow plate, Seed lantern, exposures were made only through the blue and green filters, the resolution to blue light was much higher even than that given by exposure to white light. With the orthochromatic plate the same was true, the resolving power to the blue light being higher. On the panchromatic material, the resolution was best in the blue, dropping in the green and rising again in the red but not to as great a value as given by the blue light.

The curves of Fig. 9 illustrate these changes. All exposures were developed in the same developer for the same time under controlled conditions. In all of these experiments refocusing was accomplished for each wave length

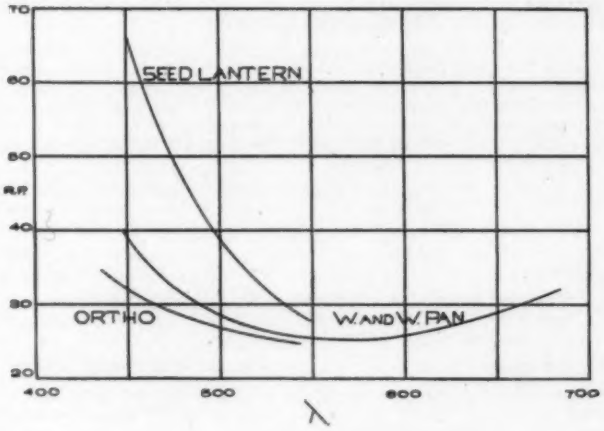


Fig. 9

by means of the slow motion screw already mentioned. This was done photographically by trial and error, thus correcting for the chromatic aberration of the lens used.

Summary

Photographic resolving power has been studied by the fan grating test to ascertain the effect of exposure, development, developer used, and the wave length of the incident light. It was found that resolution is, in general, extremely sensitive to time of exposure, over and under-exposure being decidedly detrimental. It also depends upon the time of development and upon the reducing agent employed. If two plates of the same emulsion, exposed equally and under the same conditions, are developed in different developers, the values of resolving power obtained vary widely, the greatest deviation from the maximum value being of the order of forty per cent, with the developers tested. Thus, resolution is not a definite property of an emulsion but is dependent also upon its treatment in the photographic process. With regard to the effect of the wave length of the incident light, resolution was best for light of the short wave lengths with a decided minimum in the green, increasing again in the red, though not to as high a value as with blue light.

Research Laboratory
Eastman Kodak Company
Rochester, N. Y.
November, 1917

APPLIED OPTICS ABROAD

Both France and England have recently planned considerable developments along the lines of technical optics. In France, according to a recent article in *La Nature* (abstracted in *Science* in June 25, page 609), there is to be established this year in Paris (1) a college of optics, (2) a central optical laboratory, (3) a trade school and (4) a journal devoted to articles on applied optics; forming altogether an Institute of Applied Optics under the direction of Professor Charles Fabry.

The College of Optics is intended to provide a thorough theoretical and practical training for those intending to teach or work in applied optics, and to promote among its students a taste for research in both theoretical and practical optics. The students in the college of optics would come from the educated classes; army and navy officers, students and ex-students of the universities and technical colleges, astronomers, illuminating engineers, manufacturers of optical instruments, eye specialists and those intending to specialize in physiological optics. It is intended that the courses of lectures should be supplemented by special lectures on all modern optical questions.

The Optical Laboratory is intended primarily to provide means for research in connection with theoretical optics and to provide the highest grade instruments for testing optical manufactures. It will also provide ordinary laboratory facilities for the students of the college of optics.

The Trade School would give a three years course to young students and apprentices in grinding and polishing lenses and prisms, and in the construction and fitting up of optical instruments.

It is proposed that the Institute publish perodical Transactions dealing not only with the work of the college of optics, the laboratory and the trade school, but optical matters in general. The publication of the Transactions of the Institute is said to be already assured.

It is planned to begin the courses of work in the Institute in the school year 1917-18.

In England, the London County Council has established a separate Department of Technical Optics at the Imperial College of Science and Technology, acting on the advice of its Technical Optics Committee. (*Nature*, May 24, 1917, page 257.)

This newly created department is under the direction of Prof. F. J. Cheshire, President of the Optical Society of Great Britain.

Work in the new department is to be carried on during the coming college year. Among other courses scheduled are two of 12 lectures each by Prof. A. E. Conrady on "The Designing and Computing of Telescope Systems."

There has recently been formed in Great Britain a Society of Glass Technology. The first numbers and reprints of the Transactions of the Society of Glass Technology have recently appeared.

ABSTRACTS

THE FUNDAMENTAL PRINCIPLES OF GOOD LIGHTING. P. G. Nutting, Journal of the Franklin Institute, 184, pp. 287–302, March 1917. Recent advances in the technical side of lighting have made available practically any desired quantity and quality of illumination. The time is ripe for the technical investigator to determine just what lighting is most desirable. This paper, based largely on experimental work on vision, is an attempt to outline the fundamental principles upon which the illuminating engineer may determine in any particular case what is the proper illumination to secure the desired quality of seeing at a minimum of expense.

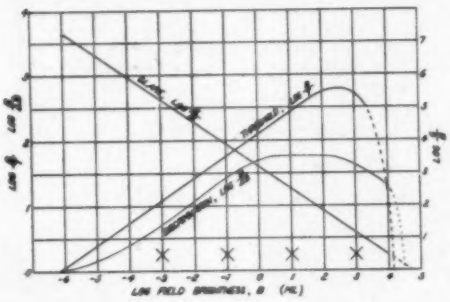
The first fundamental relation is that between sensation and light stimulus in the steady state. Since sensation can not be directly measured, the sensibility corresponding to each general level of sensitizing field brightness is measured and the sensation determined by integrating that sensibility. Four kinds of sensibility are defined and measured. (1) Threshold sensibility, the reciprocal of the instantaneous threshold or brightness just perceptible, in millilamberts, corresponding to each brightness of sensitizing field viewed. (2) Photometric sensibility, the reciprocal of the relative difference in brightness of too closely adjacent fields when that difference is barely perceptible. (3) Flicker sensibility measured by the just noticeable difference in brightness of two brightnesses occupying the same field of view alternately. (4) Glare sensibility measured in terms of the brightness of a field just bright enough to appear glaring. Flicker sensibility is very nearly the same as photometric sensibility.

In the following table are given retinal sensibilities for each field brightness from 10^{-6} millilamberts up to 10 lamberts:

VISUAL SENSITOMETRIC DATA

Field Brightness in millilamberts	Difference Fraction	Discrimination Factor	Threshold Limit in millilamberts	Glare Limit
0.000001	(1.00)	0.017	0.00000093	20.0 millilamberts
0.00001	(0.66)	0.026	0.0000042	40.7
0.0001	0.395	0.043	0.000019	89.0
0.001	0.204	0.078	0.000087	186.0
0.01	0.078	0.220	0.00039	400.0
0.1	0.0370	0.465	0.00174	810.0
1.0	0.0208	0.830	0.0081	1.68 lamberts
10.0	0.0174	0.990	0.036	3.47
100.0	0.0172	1.000	0.28	7.25
1000.0	0.0240	0.719	2.15	14.45
10000.0	(0.048)	(0.360)	(232.0)	30.90

In the second column are the just perceptible difference fractions, in the third column the discrimination factor, the reciprocal of the preceding data reduced to maximum ordinate unity. This is a measure of the ability of the eye to distinguish details differing but slightly in brightness under various levels of general illumination. In the fourth column are given data on the instantaneous thresholds corresponding to each field brightness, and in the final column the glare limit.



In the figure are shown graphs of these data plotted as logarithmic ratios. Log glare sensibility is a linear function of log field brightness such that

$$\text{Log } G = 3.23 + 0.32 \log B$$
$$G = 1700B^{0.32}$$

Log threshold sensibility is also a linear function of log field brightness except at the highest intensities.

$$\text{Log } T = 0.66 \log B - 2.10$$
$$T = 125B^{0.66}$$

The discrimination factor varies irregularly, much as it does in the case of photographic surface. It is approximately linear in the region from 10^{-4} millilamberts up to 1 millilambert and approximately constant from 1 millilambert up to 1 lambert.

In contrasty fields of limited angular area, the sensibility varies considerably with the angular size of the field and its position relative to the axis of vision, as well as upon the general brightness of field viewed.

The rate of increase and decrease of sensibility after an abrupt change of general field brightness follows a well defined logarithmic curve and varies with the wave length of the light, the degree of contrast in field as well as upon size of field and distance from the axis. This work on vision in contrasting fields and on adaptation has been repeated and extended by Julian Blanchard and Prentice Reeves, whose papers are to be published shortly.

The seeing ability to be provided for by proper illumination is roughly in three quite distinct classes. In (1) mere outlines of large objects are to be observed as in riding along a country road at night. (2) The outlines of small objects as in reading or seeing the minutest detail which the eye is capable of resolving. As examples of class (3) may be mentioned photo-engraving, drafting, sewing and many other arts and crafts where critical judgment is required. For seeing of class (1) but little illumination is required. The light of a high full moon supplemented by perhaps a little lighting in shadows is ample, if not ideal. Glare, however, is to be carefully avoided on account of extreme sensibility of the eye at this level. Seeing of class (2) represents the vast majority of working conditions, such as reading, writing, machine work, etc. Contrasts are ample and the observation of fine detail is not required. The specification of a minimum discrimination factor, such as 0.5, is reasonable. Hence, the brightness level should be not less than 0.1 millilamberts nor over 7000 millilamberts, brightnesses, ranging from 1 to 250 millilamberts without glare within the field of view are to be chosen. For seeing of class (3) only the very best lighting conditions will prevent visual discomfort and eye strain. Fine needle work requires perhaps as high class lighting as any. Interest centers upon a mass of fine bright images formed by the needle and by the fibers of textile goods. If all the light is diffused, these images broaden and merge,

shadows disappear and vision is strained by the attempt to pick up faint contrast. The most suitable lighting is limited to nearly a single direction and that quite elevated.

Ideal lighting is not necessarily that under which the eye is subjected to no glare or no strain whatever, but that which will make possible the use of the eye under the most favorable conditions in general. Above all, we should seek to relieve our overworked and much abused eyes from the wear and tear of the bad lighting conditions which we have ourselves put upon them.

THE EFFECT OF VARIOUS PHYSICAL STIMULI ON THE PUPILLARY AREA AND RETINAL SENSIBILITY. Prentice Reeves, Journal of Ophthalmology, Otology and Laryngology, pp. 616–626, September, 1917, Vol. 23. Communication No. 52 from the Research Laboratory of the Eastman Kodak Co. As there is such a close analogy between the Kodak and the eye an extensive series of investigations are being carried on in physiological optics in this Laboratory and this paper gives a brief summary of some of the results obtained up to this time. By making a detailed study of the functions of the eye it is hoped that some information may be discovered relating to some of the new uncertain points in photography.

The eye and the photographic plate are the most widely used instruments for studying light problems, and in practically all cases the final judgments are rendered by the eye. The eye operates through a range of ten billion to one, and maintains a remarkable efficiency throughout a greater part of this extensive range. By knowing the efficiency at which the eye operates we are able to determine the accuracy to which photographic results may be determined.

In this research, one of the problems studied was the least amount of physical radiation which will give rise to a perception. This depends on several factors, the greatest of which is the adaptation of the eye. When the eye is fully adapted to absolute darkness its sensibility is the greatest, and the threshold obtained under this condition gives the absolute photochemical reactivity of the retina. When viewing a 1 mm. spot at a distance of 3 meters the minimum radiation visually perceptible was found to be 17×10^{-10} ergs per second. When the test-spot was increased to a 3 cm square and the observing distance was 35 cm a dark adapted eye could just perceive a brightness of 44×10^{-7} millilamberts, while if the eye was adapted to a brightness of the average bright sunny day the least it could see was about 2 ml. When the size of the test-spot was increased in steps from a 2 mm. square to a 12 cm square the results show the sensibility varies as the visual angle up to a value equal to the angle of the fovea, and then rises gradually as the peripheral regions are stimulated. By placing an accurately calibrated shutter before the test-spot a series of determinations was made to determine the intensity to which the stimulus should be illuminated to be perceived when exposed for a given period of time.

The distinguishing of detail under all conditions is the fundamental requirement of efficient vision, so the next step was to determine the contrast sensibility of the retina. It was found to increase with an increase in field brightness through the lower intensities, remain constant for the brightnesses of ordinary vision and drop off for higher intensities. Throughout ordinary vision, the eye is able to detect a difference of 1¾ per cent. in brightness between two adjacent fields. In the determination of glare sensibility it was found that the average eye when adapted to darkness could just tolerate a brightness of 25 ml., while if it were adapted to a bright sunny day, the eye could stand a brightness of 16,000 ml. These values give us an idea of what we can see, how well we can see and what we can comfortably endure.

When the eye is subjected to a change in the brightness of the environment, it automatically adjusts itself to the new condition. If the amount of the change is small, the eye adjusts itself practically instantaneously, but if the change is great an appreciable amount of time elapses. When changing from a given brightness to darkness, the eye adapts quite rapidly for the first few minutes, and continues to adapt slowly for several hours, although for practical purposes an equilibrium is reached after an hour. When colored light is used, we find adaptation is least and most slowly for red, then come yellow, white, green and blue. When the change is from a low to a higher intensity the change is much more rapid, and a person "gets his eyes" quicker when turning on a light than when turning out a light.

TABLE I

Brightness in Millilamberts	Diameter in Millimeters Both eyes open	One eye closed
0.0	7.4	7.5
0.00015	7.15	7.25
0.60	5.3	6.5
6.3	4.1	5.7
126.0	2.6	3.3
2000.0	2.0	2.0

Accompanying the change in sensibility just described is a change in the pupillary diameter, and while the former varies over a million to one, the latter varies only about 16 to 1. A series of photographs were taken of several subjects over a range of brightnesses from utter darkness to the reflection of direct sunlight from a white paper screen. The average diameter for dark adaptation was 8 mm., the largest diameter for any pupil was 8.6 mm., and the average minimum diameter was 2 mm. The effect of exposing one or both eyes to a given brightness is shown in the following table. The photographs were taken of the right eye in both cases, and the difference between the results in the second and third columns shows the effect of closing the left eye.

ANNUAL MEETING, OPTICAL SOCIETY OF AMERICA

The second annual meeting of the Optical Society of America will be held in Pittsburgh, Monday, December 31st. Titles and abstracts of papers to be presented at the meeting should be sent to the Chairman of the Committee on meetings and papers, Dr. P. G. Nutting, Research Laboratory of the Westinghouse Manufacturing Co., East Pittsburgh, Pa.